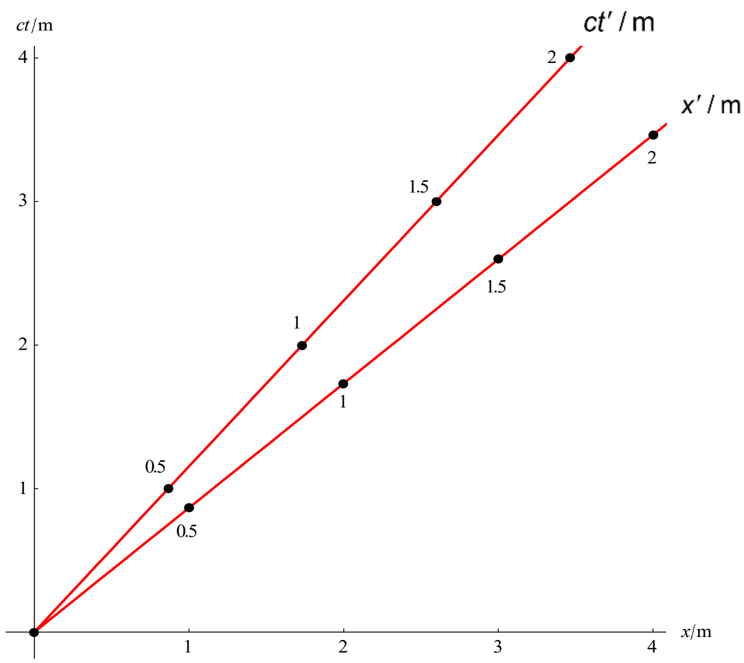
Teacher notes
Topic A

A comprehensive Relativity question.

The diagram shows the axes of a frame S (x, ct) and those of a frame S' (x', ct') that moves past S.



(a) A rod or proper length 2.0 m is at rest along the x - axis of S.

(i)	State what is meant by proper length.	[1]
(ii)	By drawing an appropriate line on the diagram, estimate the length of the rod measured in S'.	[2]
(iii)	Hence deduce that the gamma factor for the speed of S' is 2.	[1]
(iv)	Determine, in terms of c, the speed of S' relative to S.	[1]

A clock is at rest in S at $x = 0$. Event E_1 is the tick of the clock at $t = 0$ and event E_2 is the next tick of the clock at $ct = 1$ m.

(b)

(i) Suggest whether the time interval between E_2 and E_1 is a proper time interval for observers in S. [1]

(ii) By drawing an appropriate line on the diagram or otherwise, estimate the time interval between E_2 and E_1 according to S'. [2]

(c)

(i) Show, using a Lorentz transformation, that the position of E_2 in S' is $x' = -\sqrt{3}$ m. [1]

(ii) Verify that the spacetime interval between E_2 and E_1 in S and S' is the same. [2]

(d) A spacecraft left Earth on its way to a distant planet with speed 0.80*c* relative to Earth. On the way to the planet the spacecraft will go past a space station a distance of 4.0 ly from Earth, according to Earth. Two lights at the front and back of the space station light up at the same time according to the space station.

(i) Explain, without any calculations, which light turns on first according to the spacecraft. [3]

A radio signal is emitted from the spacecraft towards Earth as it goes past the space station.

Determine the time at which the signal is emitted according to

(ii) Earth, [1]

(iii) spacecraft. [1]

(iv) State the speed of the radio signal relative to Earth. [1]

Determine the time at which the signal arrives at Earth according to

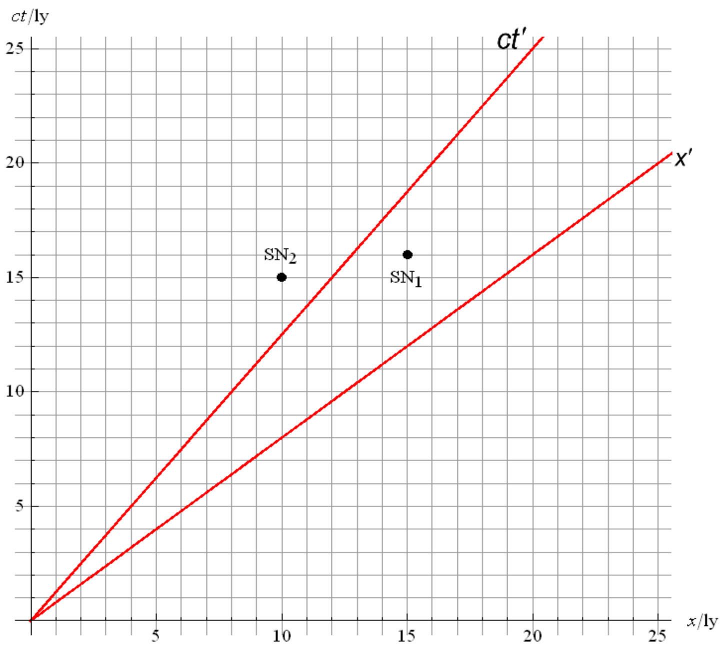
(v) Earth, [1]

(vi) spacecraft. [2]

(e) Calculate the spacetime interval between the events "signal is emitted" and "signal arrives at Earth". [1]

(f) As the spacecraft goes past the space station it emits a flare. The speed of the flare relative to the space station is 0.20*c*. and is in the same direction as the direction of motion of the spacecraft. Calculate the velocity of the flare relative to the spacecraft. [2]

(g) After the spacecraft went past the space station two supernovae happen at events SN_1 and SN_2 as shown on the spacetime diagram.



(i) State which supernova happened first according to the spacecraft. [1]

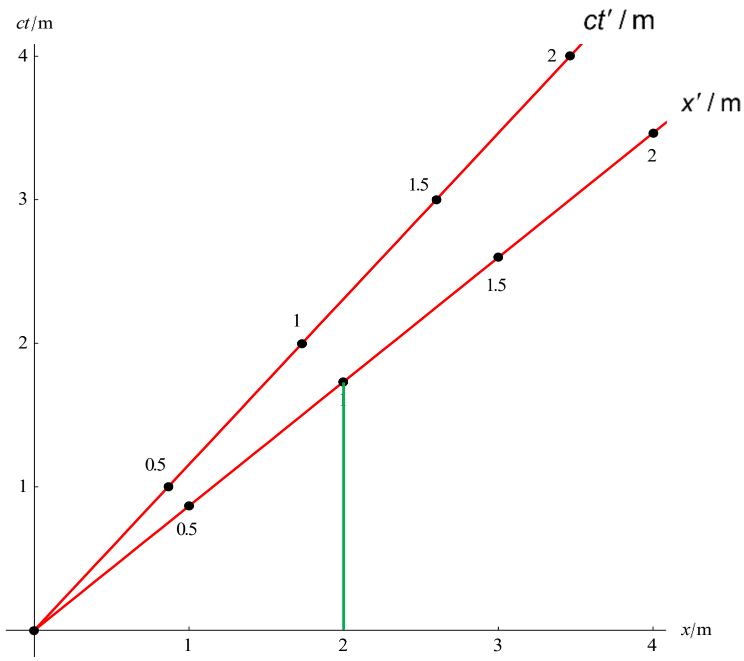
(ii) Determine the speed, relative to Earth, of a reference frame in which SN_1 and SN_2 happen at the same time. [2]

Answers

(a)

(i) The length of an object in its rest frame/when it is at rest. ✓

(ii) Vertical line through x = 2 m. ✓



Hence length = 1 m. ✓

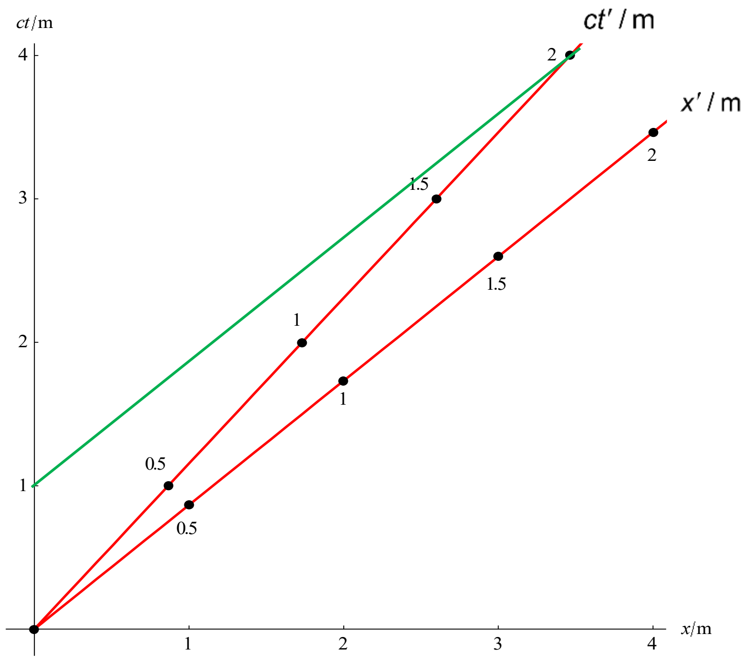
(iii) $\text{length} = \dfrac{\text{proper length}}{\gamma} \Rightarrow 1.0 = \dfrac{2.0}{\gamma}$ so gamma = 2. ✓

(iv) $\dfrac{1}{\gamma^2} = \dfrac{1}{4} = 1 - \dfrac{v^2}{c^2} \Rightarrow v = \dfrac{\sqrt{3}}{2}c$

(b)

(i) Yes, it is because the events happen at the same point in space for S. ✓

(ii) Line through ct = 1 m parallel to primed space axis. ✓



Hence $ct' = 2\text{ m}$ ✓

(c)

(i) $x' = \gamma(x - vt) = 2(0 - \frac{\sqrt{3}}{2}c \times 1) = -\sqrt{3}\text{ m}$. ✓

(ii) In S′: $\Delta s^2 = 2^2 - (\sqrt{3})^2 = 1\text{ m}^2$. ✓

In S: $\Delta s^2 = 1^2 - 0^2 = 1\text{ m}^2$. ✓

(d)

(i) An observer in the middle of the space station will receive light from the two lamps at the same time. ✓
The spacecraft observers agree that that the space station observer receives light from the two lamps at the same time. ✓
Since the space station observer is moving away from the front light, the front lamp turned on first. ✓

(ii) $t = \frac{4.0\text{ ly}}{0.80c} = 5.0\text{ yr}$. ✓

(iii) The time of travel is a proper time for the spacecraft ✓

so proper time $= \frac{\text{time}}{\gamma} = \frac{5.0}{\frac{5}{3}} = 3.0\text{ yr}$. ✓

(iv) c ✓

(v) $t = \frac{4.0\text{ ly}}{c} = 4.0\text{ yr}$. ✓

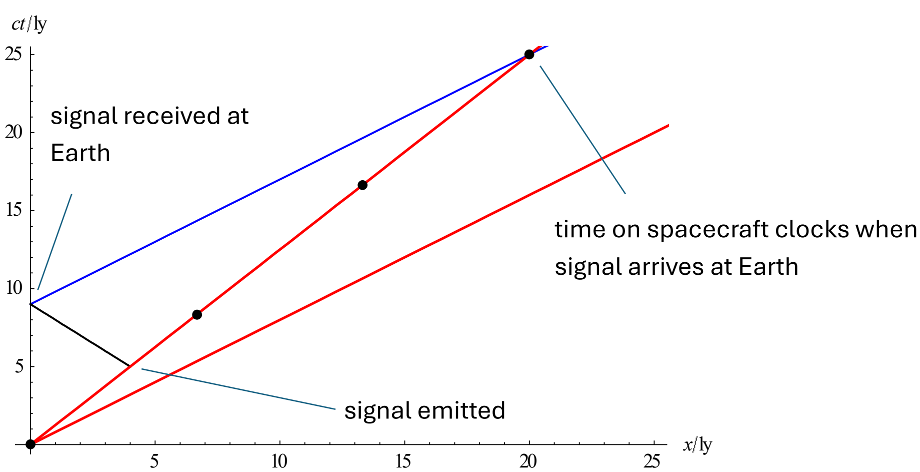
This is the time of travel for the radio signal. The Earth clock then shows 9.0 yr. ✓

(vi) The radio signal arrives at $x = 0$, $ct = 9.0$ ly. ✓

Hence $ct' = \gamma(ct - \frac{v}{c}x) = \frac{5}{3}(9.0 - 0.80 \times 0) = 15\text{ ly}$. So, the spacecraft clock shows 15 years. ✓

(Alternatively, let T be the time of travel of the signal. In this time the Earth will move a distance $0.80cT$ away from the spacecraft that considers itself at rest. The initial distance separating the spacecraft from Earth was $\frac{4.0}{\frac{5}{3}} = 2.4\text{ ly}$. Hence $0.80cT + 2.4 = cT \Rightarrow cT = \frac{2.4}{0.2} = 12\text{ ly}$. The spacecraft clock showed 3.0 yr when the signal was emitted so it now shows 3.0 + 12 = 15 yr.)

This is shown on a spacetime diagram. The dots are separated by 5 ly.



(e) The calculation is easiest in S: $\Delta s^2 = (9-4)^2 - (5-0)^2 = 0$ ✓.

(In the spacecraft frame we must find the position of the event "arrival at Earth": this is $x' = \gamma(x - vt) = \frac{5}{3}(0 - 0.80c \times 9.0) = -12\text{ ly}$. Hence $\Delta s^2 = (15-3)^2 - (-12-0)^2 = 0$.)

(f) We have $v = 0.80c$, $u = 0.20c$ and we are looking for the primed u ✓
Using the booklet formula

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{-0.60c}{1 - 0.12} = -0.68c \cdot ✓$$

(g)

(i) SN1 ✓

(ii) SN_1 and SN_2 must lie on a line parallel to the space axis of the new frame. ✓

Hence $v = \frac{1.0}{\frac{5}{c}} = \frac{c}{5}$. ✓